Introduction to Theoretical Ecology

What is theoretical ecology

Week 1 (Sept. 28, 2021)

Outline

1. **What is an ecological model?**

2. **How are ecological models created?**

3. **How to use ecological models in your study?**

4. **What will we cover in this course?**

5. **Quick review on relevant math techniques**

1. What is an ecological model?

- **What is an ecological theory?**

  An explanation of a phenomenon in the form of narratives that explain how a process works or why a pattern is observed, and have become scientifically useful when expressed in a logical structure

- **What are mathematical models?**

  Transforming the idea in narrative form into logical testable theory often involves the use of mathematics

  Equations that describe how different aspects of a system relate to one another; they are idealized and simplified versions of reality

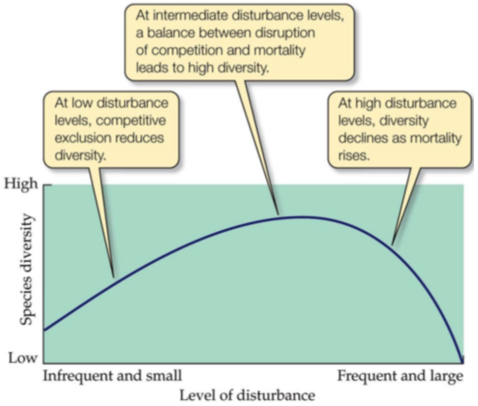
1. What is an ecological model?

- **What is an ecological theory?**

  An explanation of a phenomenon in the form of narratives that explain how a process works or why a pattern is observed, and have become scientifically useful when expressed in a logical structure

Intermediate disturbance hypothesis: fluctuations prevent competitive exclusion

Ecological phenomenon



Hypothesis / narrative

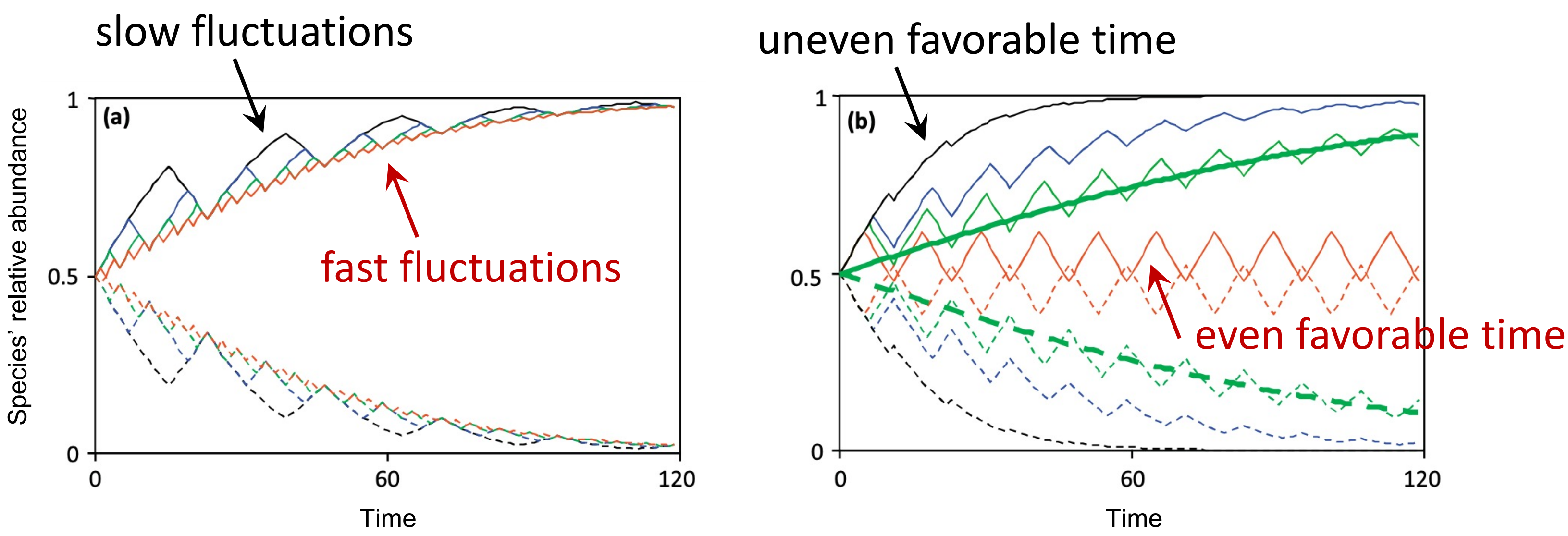
$$\frac{1}{N_i}\frac{dN_i}{dt} = a_i R - m_i(t)$$

Mathematical model

1. What is an ecological model?

Math provides a clearer and more objective expression of relationships, it brings to light assumptions and logical errors that may be obscured in verbal hypotheses, and it places ideas and hypotheses in concise form

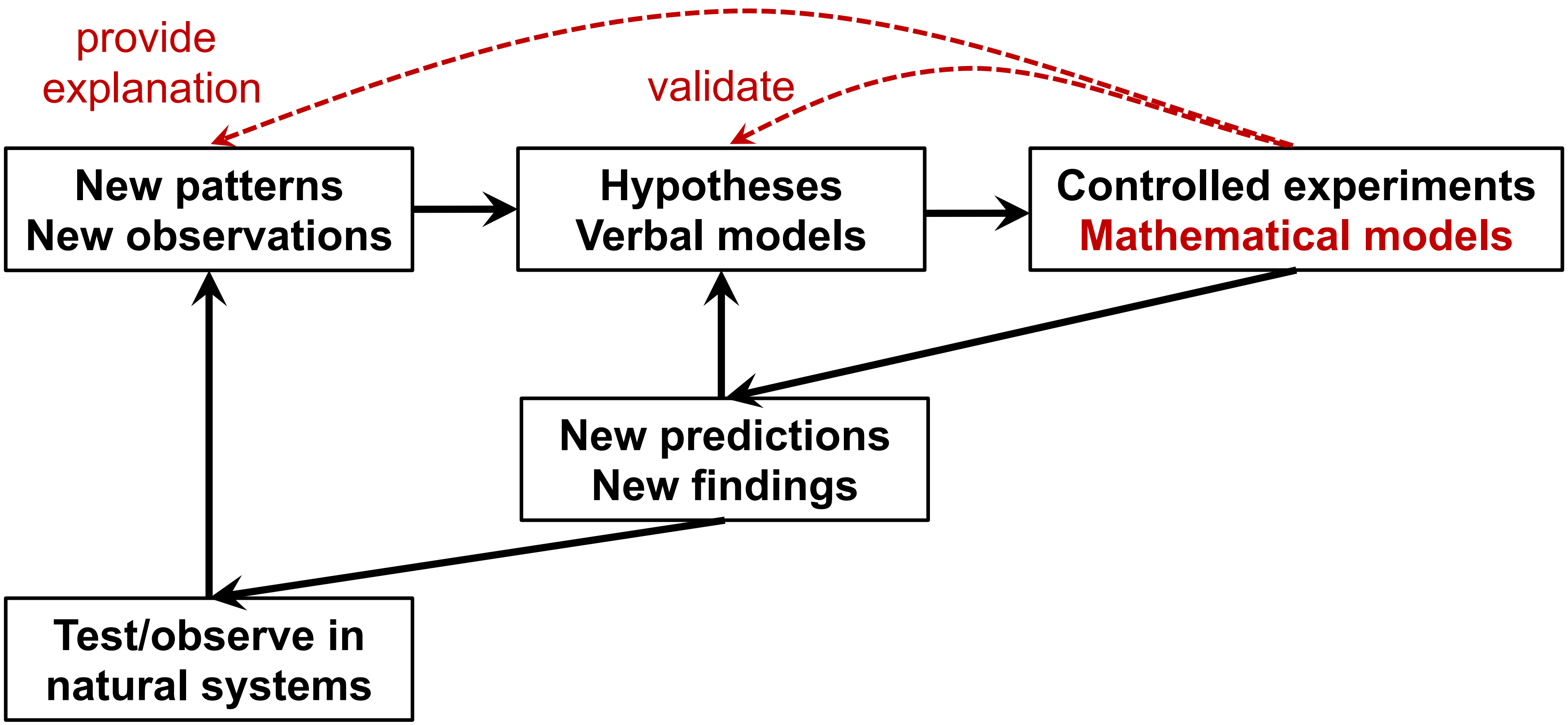
Intermediate disturbance hypothesis: fluctuations prevent competitive exclusion



Fox (2013) Treads Ecol Evol

1. What is an ecological model?

- **Continuous feedback between empirical work and model**



2. How are ecological models created?

- **Step 1: Formulate the motivating question**

  Theoreticians think of a biological question that they are interested in or a process that they wish to understand. Similar to empirical research, it can be motivated by unsolved problem or new observations

  Example: How does the number of branches of a tree change over time?

2. How are ecological models created?

- **Step 2: Determine the basic ingredients**

  Think about what entities change over time (variables), what are the biological constraints of the variable, how to deal with the passage of time, what parameters are needed?

Example: How does the number of branches of a tree change over time?
Variable: number of branches, n(t), must be a positive number

Discrete time model
(recursion equations)

$$n(t+1) = \text{some function of } n(t)$$

Continuous time model
(differential equations)

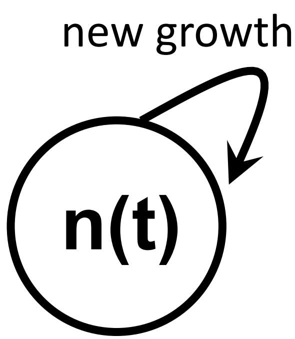
$$\frac{dn(t)}{dt} = \text{some function of } n(t)$$

2. How are ecological models created?

- **Step 3: Qualitatively describe the biological system**

  Organize the model conceptually using diagrams or tables. This makes it easier to see whether the necessary variables/parameters are included

Example: How does the number of branches of a tree change over time?
Variable: number of branches, n(t), must be a positive number



$$n(t+1) = \text{some function of } n(t)$$

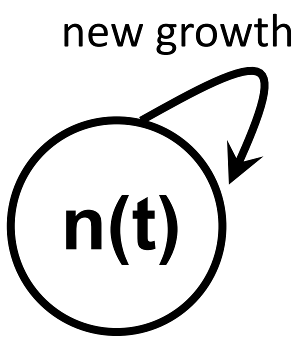
$$\frac{dn(t)}{dt} = \text{some function of } n(t)$$

2. How are ecological models created?

- **Step 4: Quantitatively describe the biological system**

  Derive the mathematical equations by tracking all of the factors that cause the variable to increase or decrease

Example: How does the number of branches of a tree change over time?
Variable: number of branches, n(t), must be a positive number



B: proportion of branches that produced new branches (unitless)

$$n(t+1) = n(t) + Bn(t)$$

$$\frac{dn(t)}{dt} = bn(t)$$

b: per capita production rate of new branches (1/time)

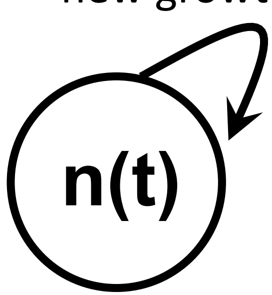
2. How are ecological models created?

- **Step 5: Analyze the model**

  Analytically: graphical analysis, stability analysis, deriving general solutions

  Numerically: simulations

Example: How does the number of branches of a tree change over time?
Variable: number of branches, n(t), must be a positive number

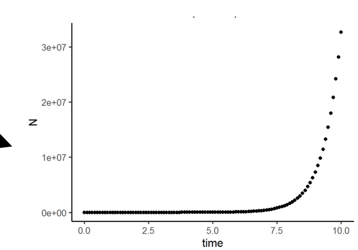


$$\frac{dn(t)}{dt} = bn(t)$$

analytical solution

$$n(t) = n_0 e^{bt}$$

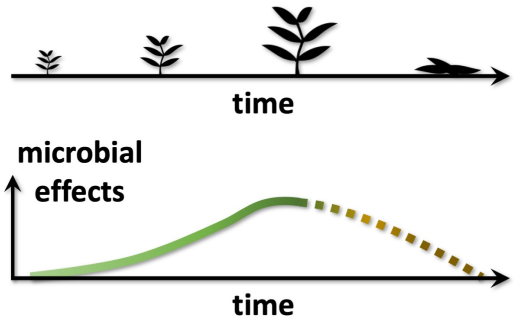
numerical solution

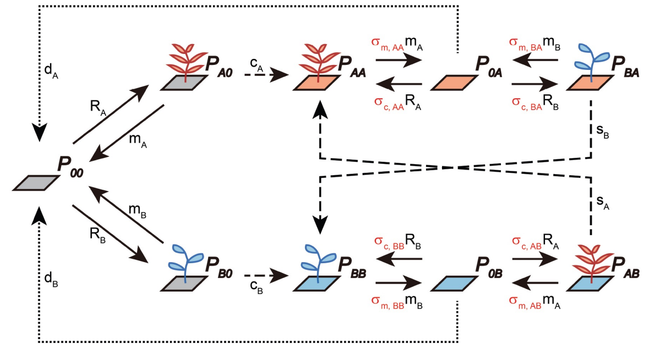


2. How are ecological models created?

- **Same steps apply to more complicated models**

Example: How do soil conditioning and decay rates affect plant competitive outcome?





$$\frac{dP_{00}}{dt} = -r_A P_{AX} P_{00} - r_B P_{BX} P_{00} + m_A P_{A0} + m_B P_{B0} + d_A P_{0A} + d_B P_{0B}$$

$$\frac{dP_{A0}}{dt} = r_A P_{AX} P_{00} - m_A P_{A0} - c_A P_{A0}$$

$$\frac{dP_{B0}}{dt} = r_B P_{BX} P_{00} - m_B P_{B0} - c_B P_{B0}$$

$$\frac{dP_{AA}}{dt} = c_A P_{A0} + \sigma_{c,AA} r_A P_{AX} P_{0A} + s_A P_{AB} - m_A P_{AA}$$

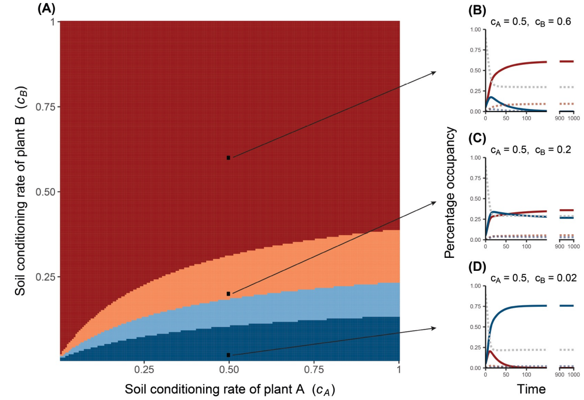
$$\frac{dP_{BB}}{dt} = c_B P_{B0} + \sigma_{c,BB} r_B P_{BX} P_{0B} + s_B P_{BA} - m_B P_{BB}$$

$$\frac{dP_{AB}}{dt} = \sigma_{c,AB} r_A P_{AX} P_{0B} - s_A P_{AB} - m_A P_{AB}$$

$$\frac{dP_{BA}}{dt} = \sigma_{c,BA} r_B P_{BX} P_{0A} - s_B P_{BA} - m_B P_{BA}$$

$$\frac{dP_{0A}}{dt} = m_A P_{AA} + m_B P_{BA} - \sigma_{c,AA} r_A P_{AX} P_{0A} - \sigma_{c,BA} r_B P_{BX} P_{0A} - d_A P_{0A}$$

$$\frac{dP_{0B}}{dt} = m_B P_{BB} + m_A P_{AB} - \sigma_{c,BB} r_B P_{BX} P_{0B} - \sigma_{c,AB} r_A P_{AX} P_{0B} - d_B P_{0B}$$

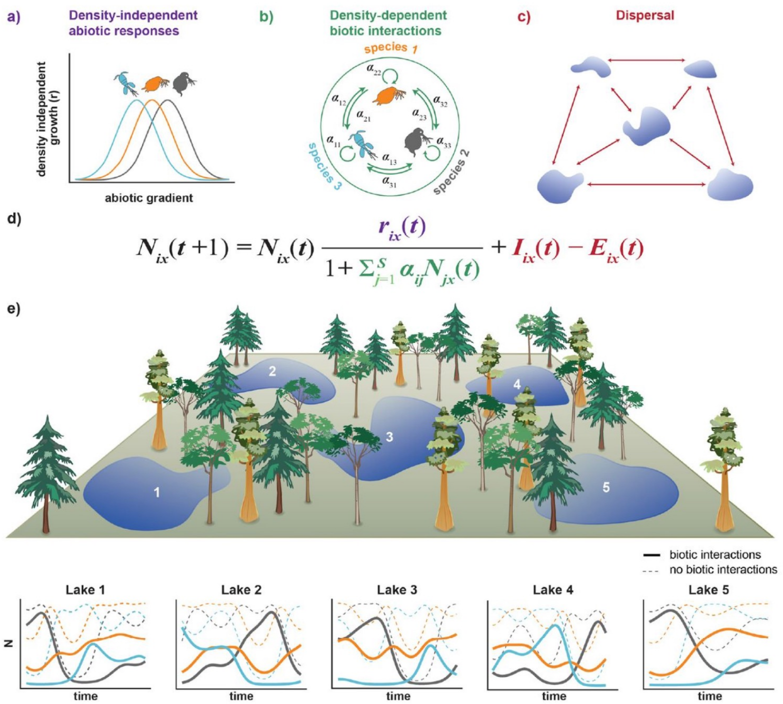


Ke & Levine (2021) Am. Nat.

3. How to use ecological models in your study?

- **Approach 1: Adopt the framework**

  Some theories offer a new way of thinking about a problem or a unification of related ideas. They reorient how we study a biological process and can help focus empirical research on a specific process or relationship



Metacommunity theory
The combined role of local and regional process

Thompson et al. (2020) Ecol. Lett.

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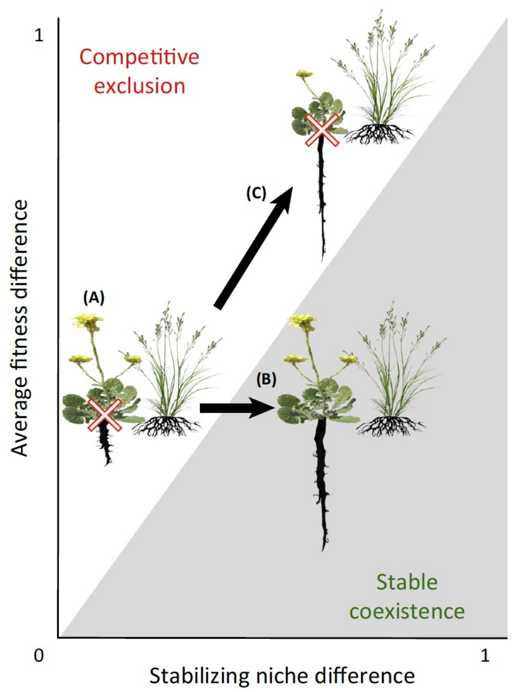


Modern coexistence theory
Species differences can both help and hinder coexistence

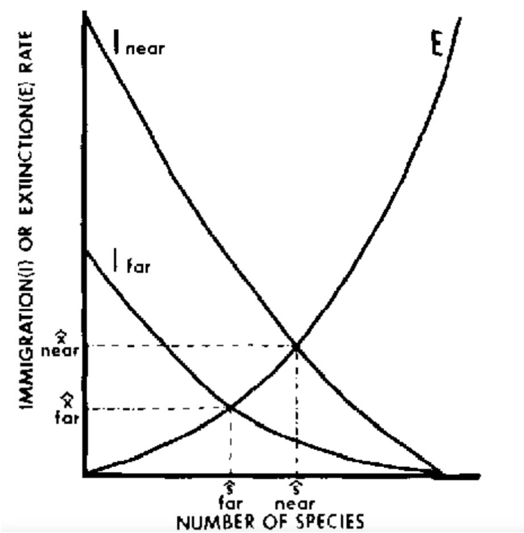
Turcotte & Levine (2016) Trends Ecol Evol

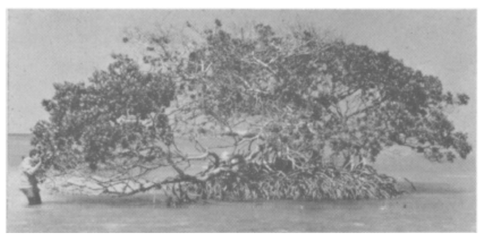
3. How to use ecological models in your study?

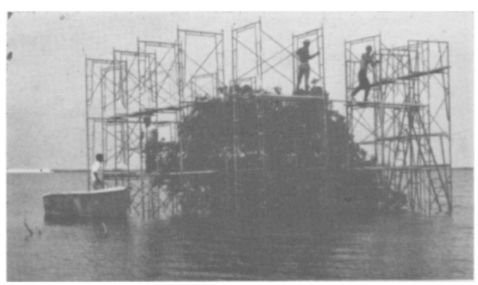
- **Approach 2: Test the predictions**

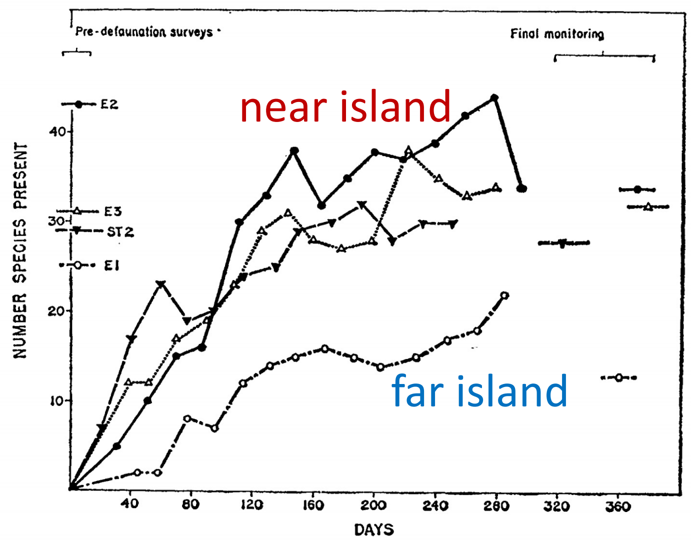
  Theoretical work often generates specific predictions, and one way to use theory is to test whether a theoretically-predicted pattern matches a pattern that manifests in a natural or experimental system

The theory of island biogeography: far islands equilibrate with lower species









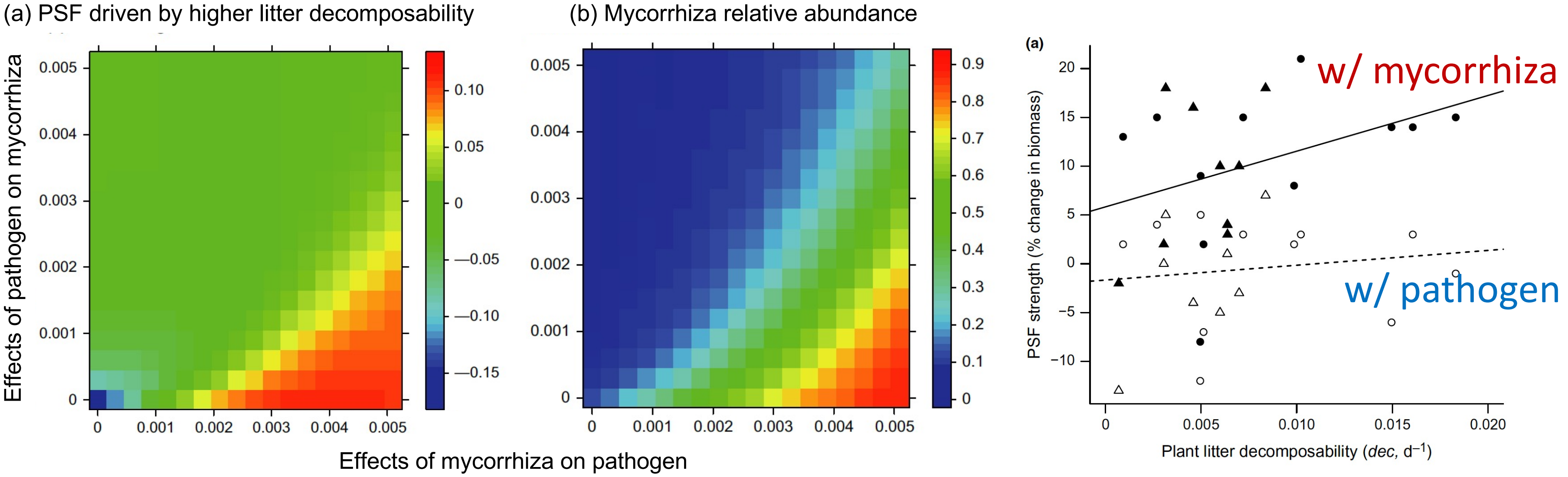
MacArthur & Wilson (1967); Wilson & Simberloff (1969) Ecology

3. How to use ecological models in your study?

- **Approach 2: Test the predictions**

  Theoretical work often generates specific predictions, and one way to use theory is to test whether a theoretically-predicted pattern matches a pattern that manifests in a natural or experimental system

Trait-based plant-soil feedback model: litter is important in mycorrhiza-dominant soil



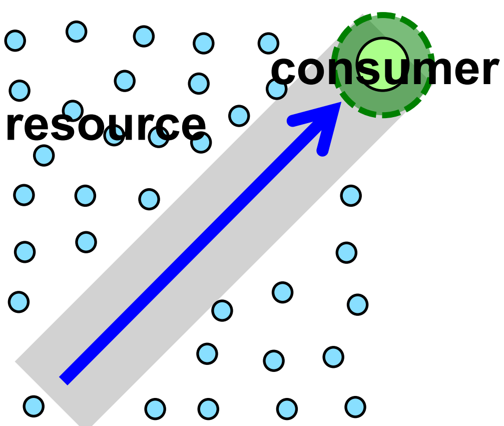
Ke et al. (2015) New Phyt

3. How to use ecological models in your study?

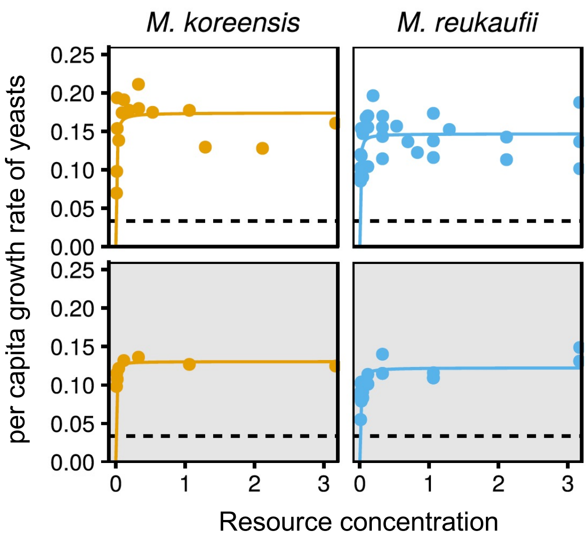
- **Approach 3: Use the equations (for model fitting)**

Some theories provide empiricist a specific equation that can be used to obtain a quantitative estimate of a biological process that is difficult or impossible to measure directly

Holling's disc equation: type II functional response for feeding rate



$$y = \frac{\overset{\text{attack rate}}{\overset{\downarrow}{a}}R}{1 + a\underset{\text{handling time}}{\underset{\uparrow}{h}}R}$$



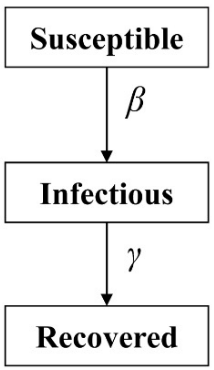
Letten et al. (2018) PNAS

3. How to use ecological models in your study?

- **Approach 3: Use the equations (for proxy calculation)**

  Some theories provide empiricist a specific equation that can be used to obtain a quantitative estimate of a biological process that is difficult or impossible to measure directly

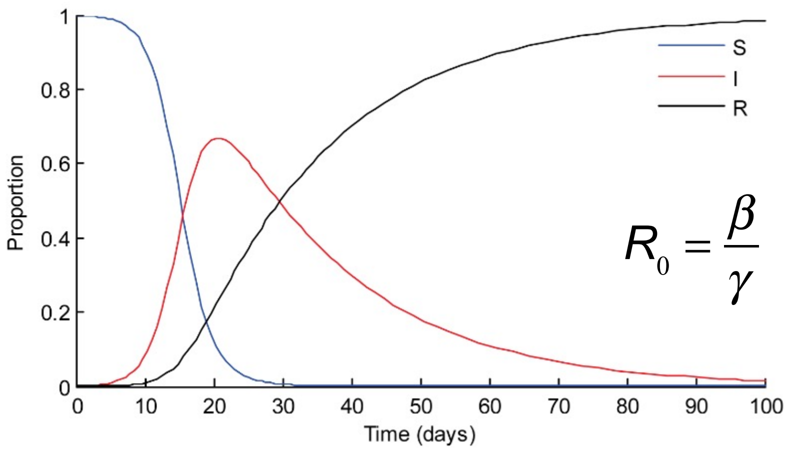
SIR models for epidemiology: R_0 for disease spread



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$



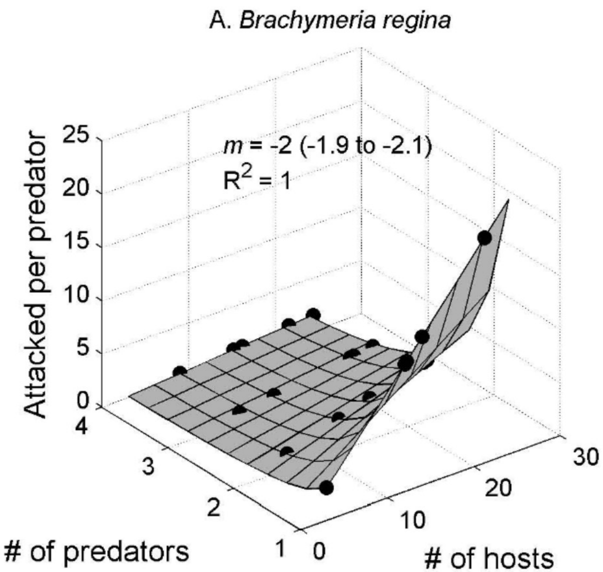
Luz et al. (2010) PLOS Neglect Trop D

3. How to use ecological models in your study?

- **Approach 4: Test model assumptions**

  Some assumptions that theoretical studies are based on may not yet be well-supported by empirical work. Empirical studies that test model assumptions can feed back to inform subsequent theoretical work

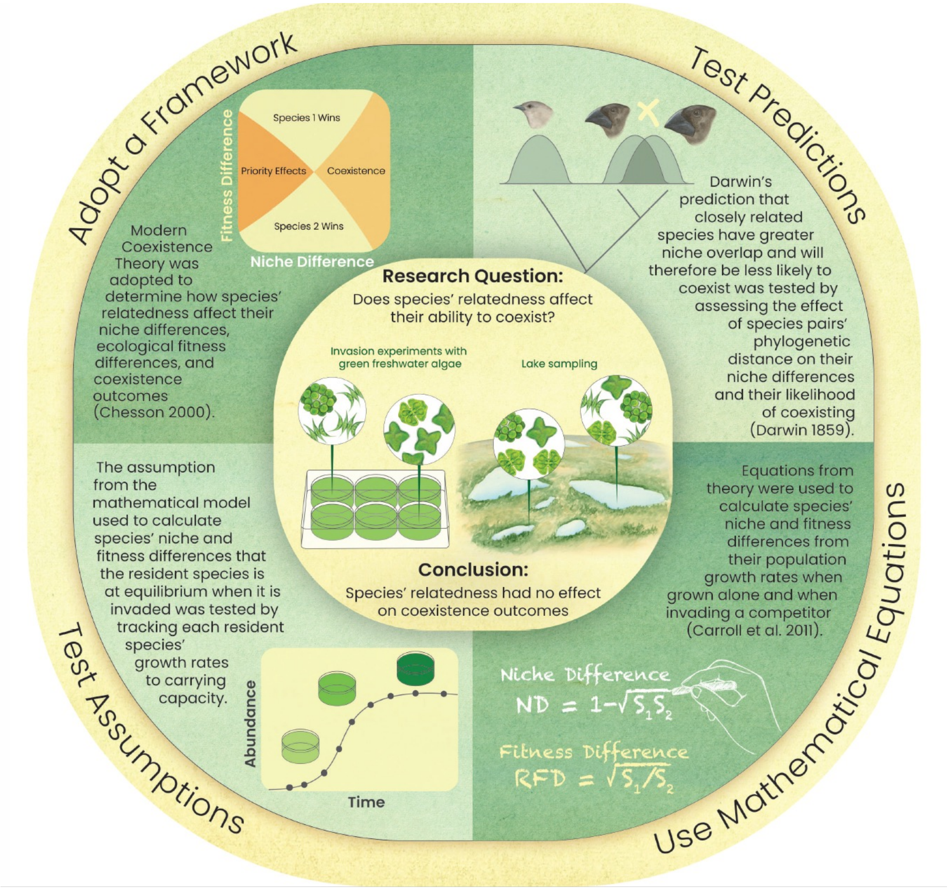
Predator-dependency in funcitonal responses



DeLong & Vasseur (2011) BMC ecology

3. How to use ecological models in your study?

- **A study can apply all at the same time, each at different stages**



Grainger et al (2021) Am Nat; Narwani et al (2013) Ecol Lett

4. What will we cover in this course?

- **Course will focus on:**

  Classic models in population and community ecology

  Analytical (pen-and-paper) and numerical (R) analyses of models

single-species population-level

multi-species community-level

Date	Lecture topic
Week 1 (28-Sept-2021)	Introduction: what is theoretical ecology?
Week 2 (05-Oct-2021)	Exponential population growth
Week 3 (12-Oct-2021)	Density-dependence and logistic population growth
Week 4 (19-Oct-2021)	Stability analysis of single population dynamics
Week 5 (26-Oct-2021)	Geometric growth and age-structured population models
Week 6 (02-Nov-2021)	Metapopulations and patch occupancy models
Week 7 (09-Nov-2021)	Lotka-Volterra model of competition: graphical analysis
Week 8 (16-Nov-2021)	Lotka-Volterra model of competition: linear stability analysis
Week 9 (23-Nov-2021)	Midterm exam
Week 10 (30-Nov-2021)	Predator-prey interactions
Week 11 (07-Dec-2021)	Mutualisms
Week 12 (14-Dec-2021)	Multispecies models of competition
Week 13 (21-Dec-2021)	Multispecies models of predation
Week 14 (28-Dec-2021)	Disease dynamics and SIR models
Week 15 (04-Jan-2022)	Ecosystem models and feedbacks
Week 16 (11-Jan-2022)	Final exam

4. What will we cover in this course?

- **Logistics:**

  Tuesday 1:20pm – 4:20pm @ Life science 3C

  Two hour lecture (blackboard) + one hour practice (laptop)

  Website

    https://genchanghsu.github.io/2021_Fall_Introduction_to_Theoretical_Ecology/

  Assignment submission via NTU Cool https://cool.ntu.edu.tw/courses/9312

  Contact us to schedule office hour

    Po-Ju Ke (pojuke@ntu.edu.tw); Gen-Chang Hsu (b04b01065@ntu.edu.tw)

  First three weeks will be online (lecture: Google meet; lab: Gather town)

    Google meet https://meet.google.com/nzd-cdjp-kbt

    Gather town https://gather.town/app/osrqFSf0a7q0I6uo/TheoreticalEcology

4. What will we cover in this course?

- **Grading:**

  Assignments (60%), midterm (15%), final (15%), participation (10%)

- **Textbooks:**

  1. Case (2000) *An illustrated guide to theoretical ecology*
  2. Gotelli (2008) *A primer of ecology 4th edition*
  3. Pastor (2011) *Mathematical ecology of populations and ecosystems*
  4. Otto & Day (2011) *A biologist's guide to mathematical modeling in ecology and evolution*

- **By the end of this course I hope you will:**

  Be familiar with the basic building blocks of ecological models

  Have the ability to analyze and formulate simple models

5. Quick review on relevant math techniques